

# Year 13 Further Mathematics - Pure Teacher

| Topic                              |  | Ref            | Ex             |
|------------------------------------|--|----------------|----------------|
| <b>Complex Numbers</b>             | <b>Exponential Form</b> <ul style="list-style-type: none"> <li>• Know and use the definition <math>e^{i\theta} = \cos \theta + i \sin \theta</math> and the form <math>z = re^{i\theta}</math></li> <li>• Multiply and divide complex numbers in exponential form.</li> </ul>  | P2.9           | 1A<br>1B       |
|                                    | <b>De Moivre's Theorem</b> <ul style="list-style-type: none"> <li>• Understand de Moivre's theorem.</li> <li>• Use de Moivre's theorem to derive trigonometric identities</li> <li>• Use de Moivre's theorem to find sums of series.</li> </ul>  | P2.8           | 1C<br>1D<br>1E |
|                                    | <b>nth roots of a complex number</b> <ul style="list-style-type: none"> <li>• Understand the nth roots of unity (solve <math>z^n = 1</math>) and their representation in an Argand diagram</li> <li>• Be able to find the nth roots of any complex number.</li> <li>• Use complex roots of unity to solve geometric problems.</li> </ul> | P2.10<br>P2.11 | 1F<br>1G       |
| <b>Complex Numbers Assessment</b>  |  |                |                |
| <b>Series</b>                      | <b>Method of Differences</b> <ul style="list-style-type: none"> <li>• Understand and use the method of differences to sum finite series</li> </ul>   | P4.4           | 2A             |
|                                    | <b>Maclaurin Series</b> <ul style="list-style-type: none"> <li>• Know how to express functions as an infinite series in ascending powers using Maclaurin series expansion.</li> <li>• Be able to find the series expansions of compound functions.</li> </ul>  | P4.5<br>P4.6   | 2B<br>2C<br>2D |
| <b>Series Assessment</b>           |  |                |                |
| <b>Further Calculus</b>            | <b>Improper Integrals</b> <ul style="list-style-type: none"> <li>• Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity.</li> </ul>  | P5.2           | 3A             |
|                                    | <b>Mean Value of a function</b> <ul style="list-style-type: none"> <li>• Understand and evaluate the mean value of a function.</li> </ul>  | P5.3           | 3B             |
|                                    | <b>Inverse Trigonometric Functions</b> <ul style="list-style-type: none"> <li>• Differentiate inverse trigonometric functions</li> <li>• Integrate rational functions and be able to choose trigonometric substitutions to integrate associated functions.</li> </ul>  | P5.5<br>P5.6   | 3C<br>3D       |
|                                    | <b>Partial Fractions</b> <ul style="list-style-type: none"> <li>• Extend integration using partial fractions to quadratics factors of the form <math>ax^2 + c</math> in the denominator.</li> </ul>  | P5.4           | 3E             |
| <b>Further Calculus Assessment</b> |  |                |                |

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|---|---|--------------|--------------------|
| <b>Further Volumes of Revolution</b>    | <b>Volumes of Revolution</b> <ul style="list-style-type: none"> <li>• Find volumes of revolution around the x-axis or y-axis for more complex functions.</li> <li>• Find volumes of revolution around the x-axis or y-axis for curves defined parametrically.</li> </ul>  | P5.1         | 4A<br>4B<br><br>4C |
|   | <b>Modelling with Volumes of Revolution</b> <ul style="list-style-type: none"> <li>• Using volumes of revolution to model real life situations.</li> </ul>  | P5.1         | 4D                 |
| <b>Volumes of Revolution Assessment</b> |   |              |                    |
| <b>Polar Coordinates</b>                | <b>Definition and Cartesian conversion</b> <ul style="list-style-type: none"> <li>• Understand and use polar coordinates <math>(r, \theta)</math></li> <li>• Convert from polar to Cartesian coordinates and vice-versa.</li> <li>• Convert equations between polar and Cartesian form.</li> </ul>  | P7.1         | 5A                 |
|   | <b>Sketching curves</b> <ul style="list-style-type: none"> <li>• Be able to sketch curves with simple polar equations, including trigonometric functions.</li> </ul>  | P2.1         | 5B                 |
|   | <b>Area enclosed by a polar curve</b> <ul style="list-style-type: none"> <li>• Find the area enclosed by a polar curve.</li> <li>• Find the area of a region enclosed between two polar curves.</li> </ul>  | P7.3         | 5C                 |
|   | <b>Tangents to polar curves</b> <ul style="list-style-type: none"> <li>• Find equations of tangents parallel to or at right angles to the initial line.</li> </ul>  | P7.3         | 5D                 |
| <b>Polar Coordinates Assessment</b>     |   |              |                    |
| <b>Hyperbolic Functions</b>             | <b>Definition and Graphs – sinh, cosh, tanh</b> <ul style="list-style-type: none"> <li>• Understand the definitions of hyperbolic functions: <math>\sinh x</math>, <math>\cosh x</math> and <math>\tanh x</math>.</li> <li>• Be able to sketch their graphs and know their domains and ranges.</li> </ul>   | P8.1         | 6A                 |
|   | <b>Inverse hyperbolic functions – arsinh, arcosh, artanh</b> <ul style="list-style-type: none"> <li>• Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges.</li> <li>• Derive and use the logarithmic forms of the inverse hyperbolic functions.</li> </ul>   | P8.3<br>P8.4 | 6B                 |
|   | <b>Identities and Equations</b> <ul style="list-style-type: none"> <li>• Prove identities and solve equations using hyperbolic functions.</li> </ul>  | P8.1<br>P8.3 | 6C                 |
|   | <b>Calculus with hyperbolic functions</b> <ul style="list-style-type: none"> <li>• Be able to differentiate and integrate hyperbolic functions.</li> <li>• Use standard results for differentiating inverse hyperbolic functions to integrate functions of that type.</li> <li>• Choose and use a suitable hyperbolic substitution to integrate certain functions.</li> </ul> | P8.2<br>P8.5 | 6D<br>6E           |
| <b>Hyperbolic Functions Assessment</b>  |   |              |                    |

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|--|---|------------------------------|----------------------|
| <b>Differential Equations</b>            | <b>Integrating Factor</b> <ul style="list-style-type: none"> <li>• Solve first order differential equations using an integrating factor.</li> <li>• Find both general and particular solutions using given boundary conditions.</li> </ul>  | P9.1<br>P9.2                 | 7A                   |
|  | <b>Second order differential equations</b> <ul style="list-style-type: none"> <li>• Solve second order homogeneous DEs (RHS = 0) by using the Auxiliary Equation.</li> <li>• Understand what to do in the case of distinct, repeated and complex roots of the auxiliary equation.</li> <li>• Solve second order non-homogeneous DEs (RHS = <math>f(x)</math>) by finding the complementary function and particular integral.</li> </ul> | P9.4<br>P9.5<br>P9.6         | 7B<br>7C<br>7D       |
|  | <b>Modelling with differential equations</b> <ul style="list-style-type: none"> <li>• Model real life situations with first order DEs.</li> <li>• Use DEs to model simple harmonic motion and relate the solution to the model.</li> <li>• Models damped and forced oscillations using DEs and interpret their solutions.</li> <li>• Model real-life situations using coupled first order DEs and be able to solve them.</li> </ul>     | P9.3<br>P9.7<br>P9.8<br>P9.9 | 8A<br>8B<br>8C<br>8D |
| <b>Differential Equations Assessment</b> |   |                              |                      |